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We have studied band-to-band optical magnetoabsorption in a semiconductor quantum wire subjected to a transverse magnetic field. The magnetic field induces a blueshift in the absorption peaks and makes the linewidth narrower. Furthermore, it quenches photoluminescence and absorption in much the same way as an electric field in the quantum confined Franz–Keldysh effect (QCFKE). We call this the quantum confined Lorentz effect (QCLE), since it is the Lorentz force skewing the electron and hole wavefunctions in the quantum wire that causes the quenching. The QCLE has an advantage over the QCFKE in that it may be observed even in quantum wires with relatively leaky barriers. The other important difference is that while the QCFKE is accompanied by a redshift in the absorption or photoluminescence peak, the QCLE is accompanied by a blueshift.

I. INTRODUCTION

The effect of a transverse electric field on the optical absorption and photoluminescence in a quantum confined system is a well-researched topic.[1] The most widely studied effect in this category is the quantum confined Stark effect (related to excitonic absorption and photoluminescence) and the quantum confined Franz–Keldysh effect (QCFKE) related to band-to-band absorption in the absence of excitonic effects and electron-hole correlation. These effects have led to the conception and demonstration of novel quantum effect optical devices such as high-speed light modulators and the self-electro-optic-device (SEED),[2] besides being utilized to study myriad aspects of the physics of quantum confined structures. Despite their remarkable success however, both the QCSE and the QCFKE have a practical shortcoming. They are often difficult to observe in material systems such as AlGaAs/GaAs heterostructures because the heterobarriers that confine electrons and holes are not high enough to prevent leakage of electrons and holes (either by tunneling or by thermionic emission) when an electric field is applied perpendicular to the heterointerfaces. Indeed, it has been pointed out that the electronic and hole states in a quantum confined system are not true bound states in the presence of an electric field since the particles can always lower their energy by tunneling out of the well.[3] A high enough electric field tilts the barriers of a quantum well thereby allowing photogenerated electrons and holes to escape. Escape from the wells reduces the effective lifetime of the electron and hole states thereby reducing the absorption strength and broadening the transitions.

To mitigate this problem, we have studied the magnetic analog of the QCFKE in a quantum wire. This is illustrated in Fig. 1. A transverse magnetic field applied in the z direction exerts a Lorentz force on photogenerated electrons and holes whose wavefunctions are skewed along the y direction. The Lorentz force acts in the same *sense* on both electrons and holes (as in the Hall effect), but since the effective masses of these two particles are different, the amount of skewing is different for them. This reduces the overlap between the electron and hole wavefunctions and results in the quenching of absorption or photoluminescence.

The advantage of using a magnetic field instead of an electric field is that the electron and hole states remain true bound states in the presence of the field. Neither particle can lower their energies by tunneling out of the well. Of course, the skewing of the wavefunctions will give rise to a self-consistent electric field whose effect can be understood by solving the Schrödinger and Poisson equations simultaneously. The self-consistent field will tilt the potential barrier slightly and give rise to a finite lifetime of the states. Since the self-consistent correction is quite small in most cases, the lifetimes of the electron and hole states are still very long. Consequently, we expect to see sharper transitions and increased quantum efficiency of radiative transitions.

In this paper, we have calculated the band-to-band absorption coefficient for a GaAs quantum wire in a magnetic field as a function of incident photon energy. In the next section, we describe the theoretical formulation, followed by results. Finally, in Sec. IV, we present the conclusions.

II. THEORY

We consider a quantum wire as shown in Fig. 1. The thickness along the z direction is so small that for the range of photon energies considered, we need to consider only the lowest electron and the highest heavy-hole subband along the z direction in calculating the absorption. However, we will consider multiple transverse subbands along the y direction (for both electrons and holes) since the width is much larger than the thickness. Electronic subbands along this direction will be labeled by the index ν_e and hole subbands (regardless of whether they are light or heavy holes) by the index ν_h.

A magnetic field is applied along the z direction which quenches the band-to-band absorption and photoluminescence. It must be pointed out that electrons and holes that are photoexcited to states at the bottom of the subbands (the Landau levels) will not experience a Lorentz force since these states have no resultant velocity (they correspond to closed cyclotron orbits). Therefore, these states will not be

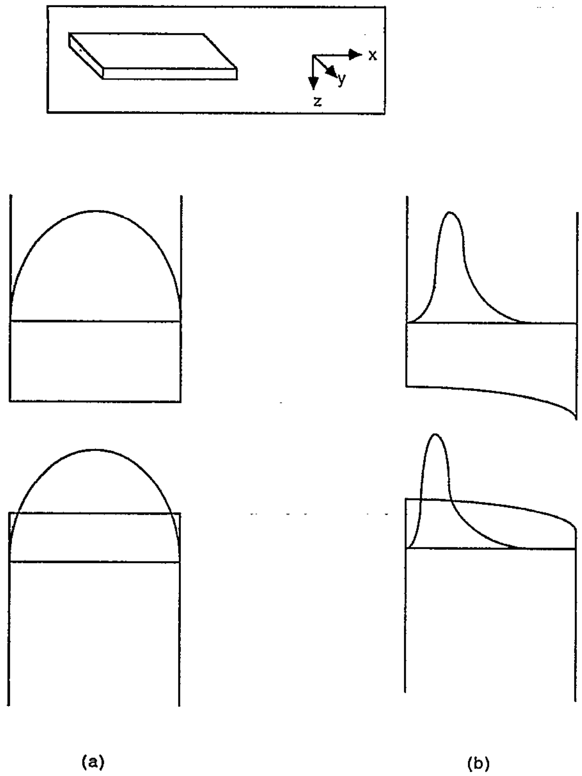


FIG. 1. Illustration of the quantum confined Lorentz effect in a quantum wire. (a) The y components of the electron and hole wavefunctions in the absence of any magnetic field, and (b) the same wavefunctions when a sufficiently strong magnetic field is applied along the z direction. The wavefunctions are skewed by the Lorentz force towards the same edge of the wire but by different amounts because of the difference in the effective masses of electrons and holes. This skewing reduces the overlap between the wavefunctions and quenches absorption or photoluminescence. The slight tilting of the potential along the y direction is not due to any external electric field, but because of the self-consistent electric field generated by the space charges. The inset shows a quantum wire with the coordinate axes.

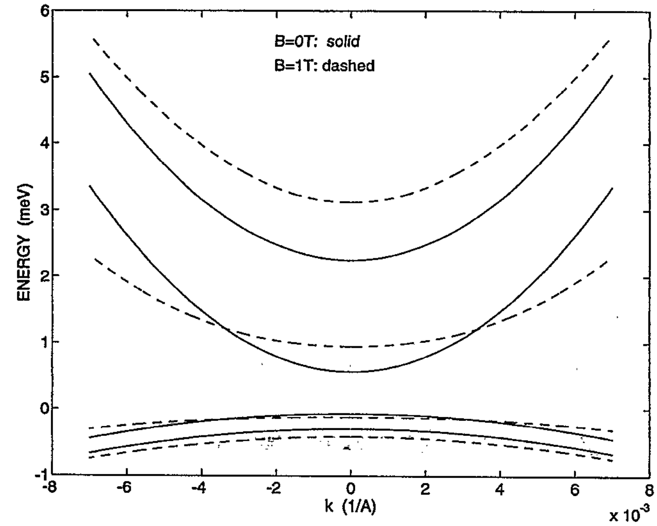


FIG. 2. Energy vs wave vector relation for electrons and heavy holes in a GaAs quantum wire of width 1000 Å. The wave vector is along the free propagation direction. The results are shown for magnetic flux densities of 0 and 1 T. In this diagram, the bulk band gap E_g plus the z-direction confinement energy is assumed to be zero for the sake of clarity.

skewed and will not exhibit the QCLE. It is only the states with a non-zero slope in Fig. 2 (the so-called "traversing states" and "skipping orbits" or "edge states" with a non-zero velocity) which will experience a Lorentz force and exhibit the QCLE. The magnetic field will also have another major effect. Since it increases the energy separation between the quantum wire subbands, it will induce a blueshift in the absorption or photoluminescence. Moreover, since the wavefunctions of electronic and hole subbands with different transverse indices are no longer orthogonal, a magnetic field can induce previously forbidden transitions between such subbands. Such transitions however will still be comparatively weak.

In explaining the physical origin of the quenching, we have invoked the concept of a classical Lorentz force acting on the electrons and holes. This is a useful picture, but it has one problem. It may appear that the electronic wavefunction will be skewed more than the hole wavefunction if the electrons are lighter than the holes. Actually, the opposite is true. The particles with the lighter effective mass will have a "stiffer" wavefunction since the energy separation between successive subbands is larger. Viewed in terms of perturbation theory, this means that a larger perturbation will be required to skew the wavefunctions of the lighter particles by a given amount. With this sole exception, the comparison with a classical Lorentz force is otherwise a useful analogy.

The absorption coefficient $\alpha(\nu_e,\nu_h,\hbar\omega)$ for transitions from a hole subband with index ν_h to an electronic subband with index ν_e as a function of the unpolarized incident photon energy $\hbar\omega$ is given by[4]

$$\alpha(\nu_e,\nu_h,\hbar\omega)=\frac{\pi q^2}{m_0\omega\sqrt{\epsilon c}}\,|P_{\nu_e,\nu_h}|^2\left|\int_0^W \psi^*_{\nu_e}(y)\psi_{\nu_h}(y)dy\right|^2 \times\sum_k \frac{1}{L}\delta[E_{\nu_e}(k)-E_{\nu_h}(k)-\hbar\omega], \quad (1)$$

where the quantity P is the momentum matrix element, k is the wave vector along the free propagation direction (x direction), W is the quantum wire width, q is the electronic charge, L is the normalizing length, ϵ is the dielectric constant, $\psi_\nu(y)$ is the y component of the electron or hole wavefunction in the νth subband, and c is the speed of light in vacuum. The above expression of course neglects excitonic effects and dressing of states (many-body effects). It also assumes the applicability of the effective mass approximation. Since we are interested in merely the band-to-band absorption, and the width of the quantum wire is several lattice spacings, this approach is permissible. Excitonic effects will be described in a future publication.

The above equation can be rewritten as

$$\alpha(\nu_e,\nu_h,\hbar\omega)=\frac{\pi q^2}{m_0\omega\sqrt{\epsilon c}}\,|P_{\nu_e,\nu_h}|^2 \times\left|\int_0^W \psi^*_{\nu_e}(y)\psi_{nu_h}(y)dy\right|^2\mathscr{T}, \quad (2)$$

where

$$\mathscr{T}=\int_{E_g+E_{\nu_e}^0+E_{\nu_h}^0}^{0} d(\Delta E_{\nu_e,\nu_h})\mathscr{D}(\Delta E_{\nu_e,\nu_h})\delta[\Delta E_{\nu_e,\nu_h}-\hbar\omega], \tag{3}$$

where

$$\Delta E_{\nu_e,\nu_h}=\Delta E_{\nu_e,\nu_h}(k)=E_{\nu_e}(k)-E_{\nu_h}(k) \tag{4}$$

and $\mathscr{D}$ is the joint density of states defined by

$$\mathscr{D}(\Delta E_{\nu_e,\nu_h})d(\Delta E_{\nu_e,\nu_h})=\frac{2L}{\pi}\,dk. \tag{5}$$

Equations (2)–(5) can be combined to yield

$$\alpha(\nu_e,\nu_h,\hbar\omega)=\frac{\pi q^2}{m_0\omega\sqrt{\epsilon c}}\,|P_{\nu_e,\nu_h}|^2\left|\int_0^W \psi_{\nu_e}^*(y)\times\psi_{\nu_h}(y)dy\right|^2\times\mathscr{D}(\hbar\omega-[E_g+E_{\nu_e}^0+E_{\nu_h}^0]), \tag{6}$$

where E_g is the material band gap and E_ν^0 is the energy at the bottom of the νth subband (measured from the bulk conduction-band edge for electrons and bulk valence-band edge for holes, plus the z-direction confinement energies).

To calculate $\mathscr{D}$, we need to know the dispersion relations (energy versus wave vector) $E_{\nu_e}(k)$ and $E_{\nu_h}(k)$ in a quantum wire subjected to a magnetic field. These, and the wavefunctions ψ, are obtained by solving the Schrödinger equation in a quantum wire subjected to a magnetic field.

The time-independent nonrelativistic Schrödinger equation describing electrons or holes in a quantum wire subjected to a magnetic field is

$$\frac{[\boldsymbol{\sigma}\cdot(\mathbf{p}-e\mathbf{A})]^2}{2m^*}\,\psi(x,y)+V(y)\psi(x,y)=E\psi(x,y), \tag{7}$$

where $\boldsymbol{\sigma}$ is the Pauli spin matrix, $V(y)$ is the confining potential in the y direction, and $\mathbf{A}$ is the magnetic vector potential. The above equation was solved[5] neglecting spin effects and assuming hardwall boundary conditions on $V(y)$. For a Landau gauge

$$\mathbf{A}=(-By,0,0), \tag{8}$$

where B is the magnetic flux density, the solution is

$$\psi(x,y)=e^{ikx}\psi_\nu(y), \tag{9}$$

where the y component of the wavefunction $\psi_\nu(y)$ obeys the equation

$$\frac{\partial^2\phi(y)}{\partial y^2}+\frac{2m^*}{\hbar^2}E\phi(y)-\left(\frac{y}{l^2}\right)^2\phi(y)+2\,\frac{y}{l^2}\,k\phi(y)-k^2\phi(y)=0 \tag{10}$$

with l being the magnetic length given by $l=\sqrt{e\hbar/B}$.

It should be noted that the electron cannot lower its energy by tunneling out of the quantum wire since the energy is not lower at $y\to\infty$ unlike in the case of an electric field.[3] Therefore, the electron and hole hybrid magnetoelectric states in a quantum wire (i.e., the eigenstates of the above equation) can be treated as true bound states.

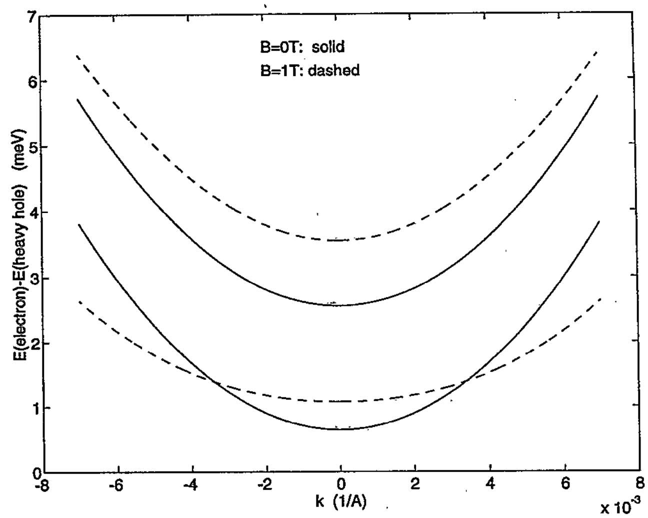


FIG. 3. The energy difference between electron- and heavy-hole subbands $\Delta E_{\nu_e,\nu_h}(k)[=E_{\nu_e}(k)-E_{\nu_h}(k)]$ as a function of the wave vector k in a 1000 Å GaAs quantum wire. The bulk band gap E_g plus the z-direction confinement energy is again assumed to be zero for clarity.

To find the wavefunction $\phi(y)$ of the magnetoelectric states, we have to solve the above equation subject to the boundary conditions

$$\phi(y=d)=\phi(y=-d)=0. \tag{11}$$

This is accomplished using a finite difference scheme as outlined in Ref. 5. The solution does not incorporate space-charge effects since the Poisson equation is not solved simultaneously. Once the eigenequation is solved, we can also find the energy-wave vector relations.[5] These relations are plotted in Fig. 2. From these relations, we obtain $\Delta E_{\nu_e,\nu_h}$ vs k as shown in Fig. 3 and finally the joint density of states $\mathscr{D}$ as a function of k or $\Delta E_{\nu_e,\nu_h}$ as defined by Eq. (5). The latter is shown in Fig. 4. Once these quantities are obtained, we can find the absorption coefficient from Eq. (6).

III. RESULTS

We consider a GaAs quantum wire of width 1000 Å along the y direction. In Fig. 5, we show the dependence of the overlap integral $|\int_0^W\psi_{\nu_e}^*(y)\psi_{\nu_h}(y)dy|^2$ on photon energy for various electron and heavy-hole subbands at a magnetic flux density of 1 T. In the absence of any magnetic field, transitions are allowed from the mth heavy-hole subband to the nth electronic subband only if $m=n$. This follows from the orthogonality of the wavefunctions for $m\neq n$. A magnetic field breaks this orthogonality by skewing the wavefunctions and therefore allows previously forbidden transitions ($m\neq n$). However, it also weakens allowed transitions ($m=n$) because it decreases the overlap from its value of unity.

It must be understood that a magnetic field alone is not sufficient to guarantee any skewing. The skewing takes place because of the Lorentz force which is proportional to the product of the magnetic flux density and the particle velocity. Consequently, wavefunctions of states at the bottom of any

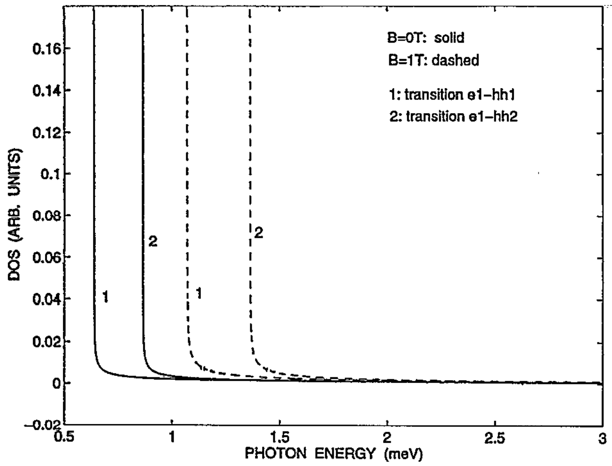
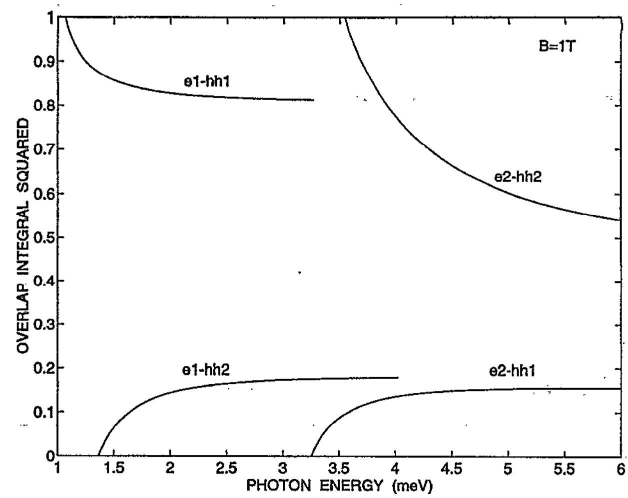
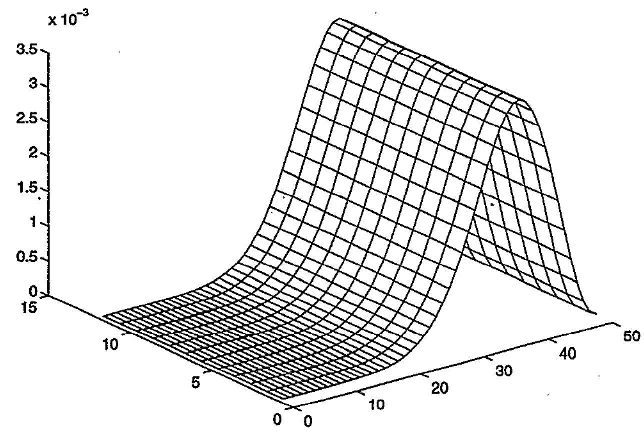


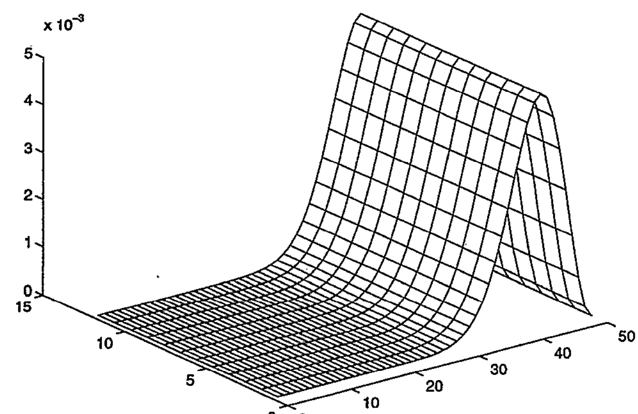
FIG. 4. The joint density of states $\mathscr{D}$ as a function of $\Delta E_{\nu_e,\nu_h}$.



(a)



(b)



(c)

FIG. 5. (a) The overlap integral $|\int_0^W \psi_{\nu_e}^*(y)\psi_{nu_h}(y)dy|^2$ as a function of photon energy $\hbar\omega$ for different electron- and heavy-hole transitions. Again the bulk band gap plus the z-direction confinement energy have been assumed to be zero. (b) The probability density $|\psi(y)|^2$ of an electronic state in the lowest electron subband 4 meV above the bulk conduction-band edge at a magnetic flux density of 1 T, and (c) the probability density of a heavy-hole state in the highest heavy-hole subband 4 meV below the bulk valence-band edge at a magnetic flux density of 1 T.

electronic subband or at the top of any hole subband is not skewed by any magnetic field since these states have a zero velocity and experience no Lorentz force. Therefore, a magnetic force has no effect on transitions that involve subband extrema. Consequently, the overlap is still unity for $m=n$ transitions (and zero for $m\neq n$ transitions) at photon energies corresponding to transitions between subband extrema. Only at photon energies away from these values does the overlap integral change in a magnetic field. This is clearly seen in Fig. 5(a).

The sharp decrease in the overlap integral (for $m=n$ transitions) with increasing photon energy in a magnetic field is a consequence of the fact that increasing photon energy excites electrons and holes to higher energy states in a subband and these states have higher velocities (see Fig. 2). Consequently they experience a stronger Lorentz force and skew more. This rapidly decreases the overlap integrals. The same physics explains why the overlap integrals increase rapidly with increasing photon energy for $m\neq n$. This is also seen in Fig. 5(a).

The rapid decrease of the overlap with increasing photon energy (for $m=n$ transitions) causes the absorption intensity for "allowed" transitions to decrease more rapidly with increasing photon energy than would have been otherwise allowed by the mere energy dependence of the density of states in a quasi-one-dimensional system. Consequently, the absorption peaks become sharper and narrower in energy. An additional cause of this narrowing is that a magnetic field makes the density of states peaks more narrow also. In the limit of an infinite magnetic field, all electron and hole states will condense into Landau levels and the density of states will become a series of δ-function spikes with ideally zero width.

The magnetoabsorption as a function of photon energy $\hbar\omega$ is shown in Fig. 6. Apart from the narrowing of the peaks with increasing magnetic field, the other interesting feature to note is the blueshift in the peaks with increasing magnetic field. Quite large blueshifts (~9 meV at 10 T) can be obtained which is larger than the thermal energy at 77 K. This shift can be utilized in magneto-optic light modulators. The

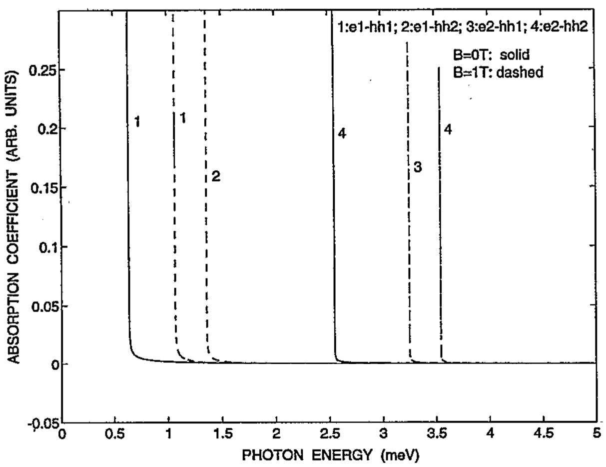


FIG. 6. The magnetoabsorption as a function of incident photon energy for various electron-hole transitions. The results are shown at 0 and 1 T. The curve labeled $e1$-$hh1$ refers to transitions between the highest heavy-hole subband and the lowest electronic subband, the curve labeled $e1$-$hh2$ refers to transitions between the second highest heavy-hole subband and the lowest electronic subband, etc.

magnitude of this shift is less than the redshift that is usually obtained in QCSE,[6] but the fact that it is blue- as opposed to a redshift is important. Producing a blueshift is considered desirable in some circumstances since it may cause lower insertion losses in optoelectronic circuits. In the past, complicated structures have been proposed to produce blueshifts in the conventional QCSE.[7] In contrast, our structure is very simple; it is a generic quantum wire.

IV. CONCLUSION

In conclusion, we have studied the magnetic field analog of QCFKE, namely the QCLE, in a quantum wire. The latter has certain advantages over the QCFKE. It can be observed in quantum wires with relatively leaky barriers. In QCFKE, an electric field causes electrons and holes to tunnel out of the well by tilting the barrier, but a magnetic field has no such effect. In fact, a magnetic field introduces additional confinement for electrons and holes trapped in cyclotron orbits. These states of course do not exhibit the QCLE since they have no resultant drift velocity, but they nonetheless exhibit a blueshift in the absorption. In principle, an arbitrarily large blueshift can be obtained by applying an arbitrarily large magnetic field without having to worry about any effect on the lifetime of the states. This may ultimately lead to an extension of the dynamic range of optical modulators employing QCSE as stronger magnetic fields become available.

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